

Reflection Symmetry and Quantized Hall Resistivity near Quantum Hall Transition

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We present a direct numerical evidence for reflection symmetry of longitudinal resistivity ρ_{xx} and quantized Hall resistivity ρ_{xy} near the transition between $\nu = 1$ quantum Hall state and insulator, in accord with the recent experiments. Our results show that a universal scaling behavior of conductances, σ_{xx} and σ_{xy} , in the transition regime decide the reflection symmetry of ρ_{xx} and quantization of ρ_{xy} , independent of particle-hole symmetry. We also find that in insulating phase away from the transition region ρ_{xy} deviates from the quantization and diverges with ρ_{xx} .

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A reflection symmetry has been recently revealed [1,2] in the transition between the quantum Hall (QH) states and the high-magnetic-field insulator: e.g., the longitudinal resistivity in the insulator and its inverse in the neighboring $\nu = 1$ QH state satisfies a relation

$$\rho_{xx}(\Delta n_\nu) = 1/\rho_{xx}(-\Delta n_\nu) \quad (1)$$

where Δn_ν is the Landau level (LL) filling number n_ν measured from the critical point $n_{\nu c}$. At the same time, the Hall resistivity remains well quantized, i.e.,

$$\rho_{xy} = h/e^2 \quad (2)$$

over the whole transition regime. Similar symmetry also holds at $\nu = 1/3 \rightarrow 0$ transition [1]. Theoretically, such a reflection symmetry has been conjectured as due to an underlying charge-flux duality [3] in the composite boson description [4,5] of the QH transitions, which is equivalent to a particle-hole symmetry in the composite fermion description [6]. Indeed, at $\nu = 1 \rightarrow 0$ transition, one can easily understand (1) if a particle-hole symmetry in the fermion description is assumed in the lowest Landau level (LLL), and with using (2). But in a general situation, disorders do not necessarily retain the particle-hole symmetry of the LLL in a microscopic Hamiltonian of non-interacting fermions in the presence of magnetic field. It is not clear theoretically whether the reflection symmetry can still remain there, even though the experiments [1,2] have clearly shown that critical point floats away from the LLL center without affecting the reflection symmetry.

The quantization of Hall resistivity (2) beyond the QH plateau regime is also nontrivial. A quantized Hall insulator has been discussed based on a phase-incoherent network model [7-9], and it has been also found [8] that ρ_{xy} may become divergent if the quantum interference is taken into account. Whether the quantization of ρ_{xy} in the transition region is a classical behavior due to the absence of quantum interference at a finite temperature as discussed in those approaches or it is a universal behavior related to the property of quantum critical point [10-12] of the QH state to insulator transition needs an independent clarification.

In this Letter, we present a direct numerical evidence for the reflection symmetry (1) in the $\nu = 1 \rightarrow 0$ transition. A quantized Hall resistivity in the transition region is also obtained in accord with (2). In particular, all these results persist even when disorders are strong enough such that Landau level (LL) mixing becomes important. It suggests that the underlying particle-hole symmetry in the LLL is not crucial to the reflection symmetry as well as the quantization of the Hall resistance. These properties are found to be related to a universal scaling of conductances near the transition. The scaling functions of σ_{xx}^2 and $\sigma_{xy} * (1 - \sigma_{xy})$ (in units of e^2/h) are found to be equal to each other as a universal curve which is symmetric with regard to the critical point, independent of sample sizes and disorder strengths. Furthermore, beyond the transition regime we find that ρ_{xy} starts to deviate from the quantized value and eventually diverges as $\rho_{xx} \rightarrow \infty$ in insulating regime. To test the robust of the reflection symmetry, we also study the case where the higher QH plateaus ($\nu > 1$) are already destroyed at strong disorders, with only $\nu = 1$ QH plateau left. This situation has been realized in a tight-binding lattice model [13-15] and there one finds another $\nu = 1 \rightarrow 0$ transition on the high filling number side [14]. While the reflection symmetry and the Hall resistance quantization are still robust near the $\nu = 1 \rightarrow 0$ transition on the low-filling-number side, they *disappear* at the transition on the high-filling side, suggesting that the existence of the QH plateau(s) *above* the Fermi energy may be crucial for both the reflection symmetry and the quantization of the Hall resistance.

We use a tight-binding Hamiltonian of non-interacting electrons:

$$H = - \sum_{\langle ij \rangle} e^{ia_{ij}} c_i^\dagger c_j + H.c. + \sum_i w_i c_i^\dagger c_i,$$

where the hopping integral is taken as the unit, and c_i^\dagger is a fermionic creation operator with $\langle ij \rangle$ referring to two nearest neighboring sites. A uniform magnetic flux per plaquette is given as $\phi = \sum_{\square} a_{ij} = 2\pi/M$, where the summation runs over four links around a plaquette. In the following we mainly focus on $M = 8$ case, while

weaker fields with larger M are also checked. w_i is a random potential with strength $|w_i| \leq W/2$, and the white noise limit is considered with no correlations among different sites for w_i . In the weak disorder limit, the mixing between Landau levels can be neglected so the disorder effect in the LLL still approximately respects the particle-hole symmetry. With the increase of W , Landau levels start to mix together such that the definition of “holes” is no longer meaningful in the LLL and the particle-hole symmetry is then removed. In the following, we mainly study $W = 1$ and $W = 4$ which represent these two limits. We also focus on the $\nu = 1 \rightarrow 0$ transition which happens at the step of the Hall conductance σ_{xy} between e^2/h and 0. σ_{xx} at the critical point scales onto a constant value $\sigma_{xx} = (0.50 \pm 0.02)e^2/h$ independent of disorder or magnetic field strength in agreement with earlier work [11,12]. Here σ_{xx} is calculated using Landauer formula [16] for square sample size $L \times L$ with leads and averaged over random disorder configurations to obtain a required statistical error (less than 2%). It has been checked by us that σ_{xx} calculated in this way agrees with the conductance calculated from Thouless number [17] as long as they are scaled to the same value at the critical point. Hall conductance σ_{xy} is calculated by using Kubo formula (at least 2,000 configurations are taken for the largest sample sizes).

The quantization of the Hall resistivity is clearly shown in Figs. 1a and 1b near the $\nu = 1 \rightarrow 0$ transition at $M = 8$. Here ρ_{xy} is plotted as a function of σ_{xy} at different sample widths: $L = 8, 16, 24$ and 32 , with $W = 1$ and $W = 4$, respectively. In such a transition regime, σ_{xy} changes from 1 to 0 (in units of e^2/h), while ρ_{xy} stays almost constant in the transition region at the quantized value 1. Such a quantization remains in spite of the change of the particle-hole symmetry from $W = 1$ to $W = 4$: the density of states, $D_L(\epsilon)$, is shown in the inserts of Figs. 1a and 1b, and in contrast to $W = 1$ where the LLL is well separated from higher LL's by a gap, two lowest LLs are mixed at $W = 4$ where the particle-hole symmetry is obviously absent.

The reflection symmetry of the longitudinal resistivity is shown in Fig. 2 at $W = 1$ and $W = 4$, respectively. Here the resistivity is plotted as a function of $\Delta\epsilon$, i.e., the Fermi energy ϵ measured from the critical point ϵ_c . As shown in Fig. 2, $\rho_{xx}(\Delta\epsilon)$ and $1/\rho_{xx}(-\Delta\epsilon)$ are right on top of each other, demonstrating the reflection symmetry of ρ_{xx} at a sample width $L = 32$. In both cases, the reflection symmetry is exhibited over a very broad region: ρ_{xx} changes more than one order of the magnitude in the insulating region similar to the experimental observation [1,2] (the region with reflection symmetry generally grows with the sample size L in our calculation).

The quantization of the Hall resistance implies a semi-circle law:

$$\sigma_{xx}^2 + (\sigma_{xy} - \frac{1}{2})^2 = \frac{1}{4} \quad (3)$$

or $\sigma_{xx}^2 = \sigma_{xy} * (1 - \sigma_{xy})$, which was previously obtained based on a semiclassical treatment of the QH edge states [9]. To make sure if this effect is an intrinsic property of a quantum critical point instead of a finite-size effect, we have checked the scaling behavior of both σ_{xx} and σ_{xy} . It has been well established [11] that $\nu = 1 \rightarrow 0$ transition should satisfy a one-parameter scaling law: both σ_{xx} and σ_{xy} at different sample sizes can be scaled onto a scaling curve if plotted as a function of L/ξ at fixed disorder strength. Here ξ is the thermodynamic localization length, $\xi = (\epsilon_0/|\epsilon - \epsilon_c|)^\alpha$ with $\alpha \approx 2.3$. But it was expected that the shape of the scaling curve may, more or less, depend on disorder potential. By contrast, we find that the scaling of conductances here is actually universal and the data at different W 's all collapse onto a universal curve if we choose a right energy scale ϵ_0 at different W 's. For example, as shown in Fig. 3a, σ_{xx} at $W = 1$ and $W = 4$ for $L = 128$ and $L = 160$ can be scaled onto the same scaling function of $L^{1/\alpha}\Delta\epsilon/\epsilon_0$ by choosing $\epsilon_0(W = 4) = 3.57$ with $\epsilon_0(W = 1)$ chosen as the unit. Notice that the data shown in Fig. 3a are symmetric about its critical point ϵ_c as shown in the insert. Similar behavior is also found for σ_{xy} . In addition to the universal scaling of σ_{xx} and σ_{xy} , we also found that the scaling curve of σ_{xx}^2 for sample sizes from $L = 24, 32, 64, 128$ to 160 coincides with the scaling curve of $\sigma_{xy} * (1 - \sigma_{xy})$ in Fig. 3b, independent of the disorder strength W . [Note that while the maximum width for σ_{xx} calculation reaches $L = 160$, the largest sample width attainable for σ_{xy} is much smaller ($L = 48$) in the present work and in literature [12].] This scaling relation guarantees the semi-circle law (3) and the exact quantization of ρ_{xy} in the whole $1 \rightarrow 0$ transition region. Thus we find that both the exact quantization of ρ_{xy} and the reflection symmetry of ρ_{xx} are the consequences of a universal scaling satisfied by the conductances which is independent of the details of the underlying model like the particle-hole symmetry and the lattice effect (we have checked weaker magnetic fields with $M = 16$ and 24 and found essentially consistent results).

It is noted that in the above numerical calculations the resistances and conductances are shown to be scaling functions with regard to the scaling variable ξ/L around the critical point. Here the zero-temperature transition is driven by Fermi energy with $\xi = (\epsilon_0/|\epsilon - \epsilon_c|)^\alpha$. On the other hand, experiments have been done at finite temperature where the thermodynamic sample size is cut-off by a finite length scale L_{in} representing the de-phasing effect such that the scaling variable becomes ξ/L_{in} . When magnetic field instead of the Fermi energy is tuned in experiment, one has $\xi = \xi_0|B - B_c|^{-\alpha} \propto |n_\nu - n_{\nu c}|^{-\alpha}$. Thus, by using such a scaling variable the reflection symmetry found in our numerical calculations is related to the observed one in experiments as given in (1).

We have also checked the behavior of ρ_{xy} vs. ρ_{xx} in whole regime. As shown in Fig. 4, the quantized ρ_{xy}

persists from the QH plateau regime with $\rho_{xx} < 1$ to the transition regime with ρ_{xx} as large as 5 which is comparable to the experimental data. [1,2] Deep into the insulating phase with $\rho_{xx} \rightarrow \infty$, ρ_{xy} becomes divergent too. The latter behavior is consistent with the result obtained by the network model calculation with including quantum interference. [8]

Finally, we have considered a case in which the reflection symmetry of ρ_{xx} and the quantization of ρ_{xy} near the $\nu = 1 \rightarrow 0$ transition regime disappear. It has been previously shown that in the tight-binding lattice model the higher QH plateaus can be destroyed first at strong disorders while the lowest plateau remains robust [14]. In this case, there exists a second $\nu = 1 \rightarrow 0$ transition between the QH state to a high filling insulator at a critical disorder W_c for a given Fermi energy. At $W < W_c$, ρ_{xy} stays quantized in the $\nu = 1$ plateau regime. But beyond W_c , ρ_{xy} starts to deviate from the quantization and continuously drop from the quantized value approaching to the classical value $\rho_{xy} = h/e^2 n_\nu$, while the reflection symmetry of ρ_{xx} is no longer present. Thus, the well-defined Hall plateau(s) above the Fermi energy may be a crucial condition for both ρ_{xy} quantization and the reflection symmetry of ρ_{xx} to occur. This second transition has been related [14] to the experimentally observed $\nu = 1 \rightarrow 0$ transition at weak magnetic field side, and detailed properties will be discussed elsewhere.

We conclude that the reflection symmetry of resistance and the quantization of Hall resistance are intrinsically present in the $\nu = 1 \rightarrow 0$ transition in a two-dimensional non-interacting electron system at strong magnetic field. Our calculations suggest that they are determined by a universal scaling of conductances which is independent of the particle-hole symmetry in the LLL. The present numerical results explain the recent experimental measurements and provide very useful insight for further theoretical studies.

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Fig. 1 The quantization of the Hall resistivity near the $\nu = 1 \rightarrow 0$ transition: ρ_{xy} versus σ_{xy} at sample width $L = 8$ (*); 16 (+); 24 (•) and 32 (◊) at disorder strength $W = 1$ (a) and $W = 4$ (b). The inserts show the density of states $D_L(\epsilon)$; ϵ_c denotes the critical point.

Fig. 2 The reflection symmetry of ρ_{xx} : $\rho_{xx}(\Delta\epsilon)$ and $1/\rho_{xx}(-\Delta\epsilon)$ coincide at $W = 1$ and $W = 4$ with sample width $L = 32$ ($\Delta\epsilon = \epsilon - \epsilon_c$).

Fig. 3 (a) σ_{xx} as a scaling function of $L^{1/\alpha}\Delta\epsilon/\epsilon_0$. Here $\alpha = 2.3$ and $\epsilon_0 = 3.57$ at $W = 4$ (ϵ_0 at $W = 1$ is chosen as the unit). The insert shows that σ_{xx} is a symmetric function of $\Delta\epsilon$ around the origin ($L = 128$ and $W = 4$). (b) σ_{xx}^2 and $\sigma_{xy} * (1 - \sigma_{xy})$ collapse onto a scaling function of $L^{1/\alpha}\Delta\epsilon/\epsilon_0$ with $L = 24, 32, 64, 128$, and 160, and $W = 1, 4$.

Fig. 4. ρ_{xy} vs. ρ_{xx} in whole regime at fixed $W = 4$.

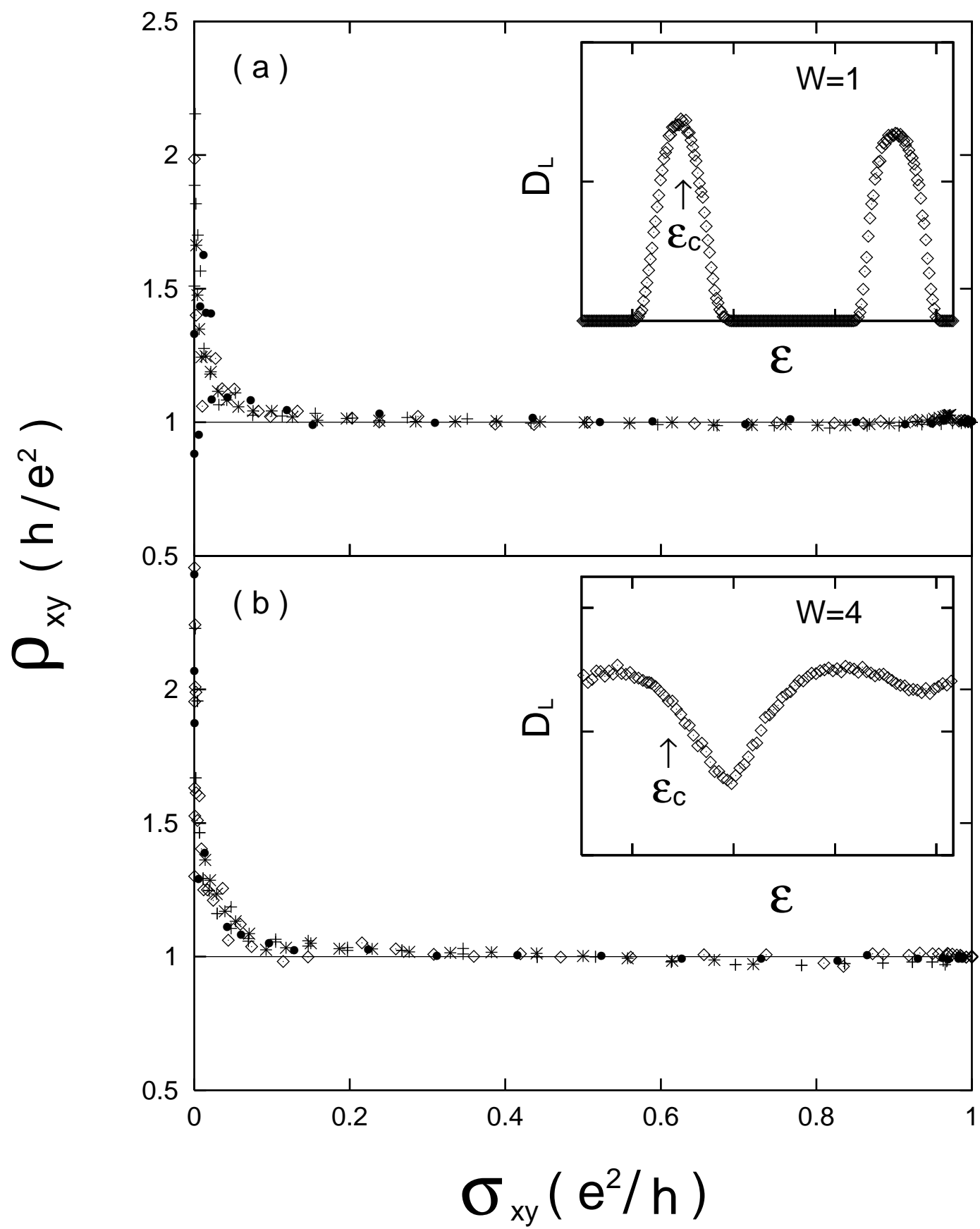


Fig. 1

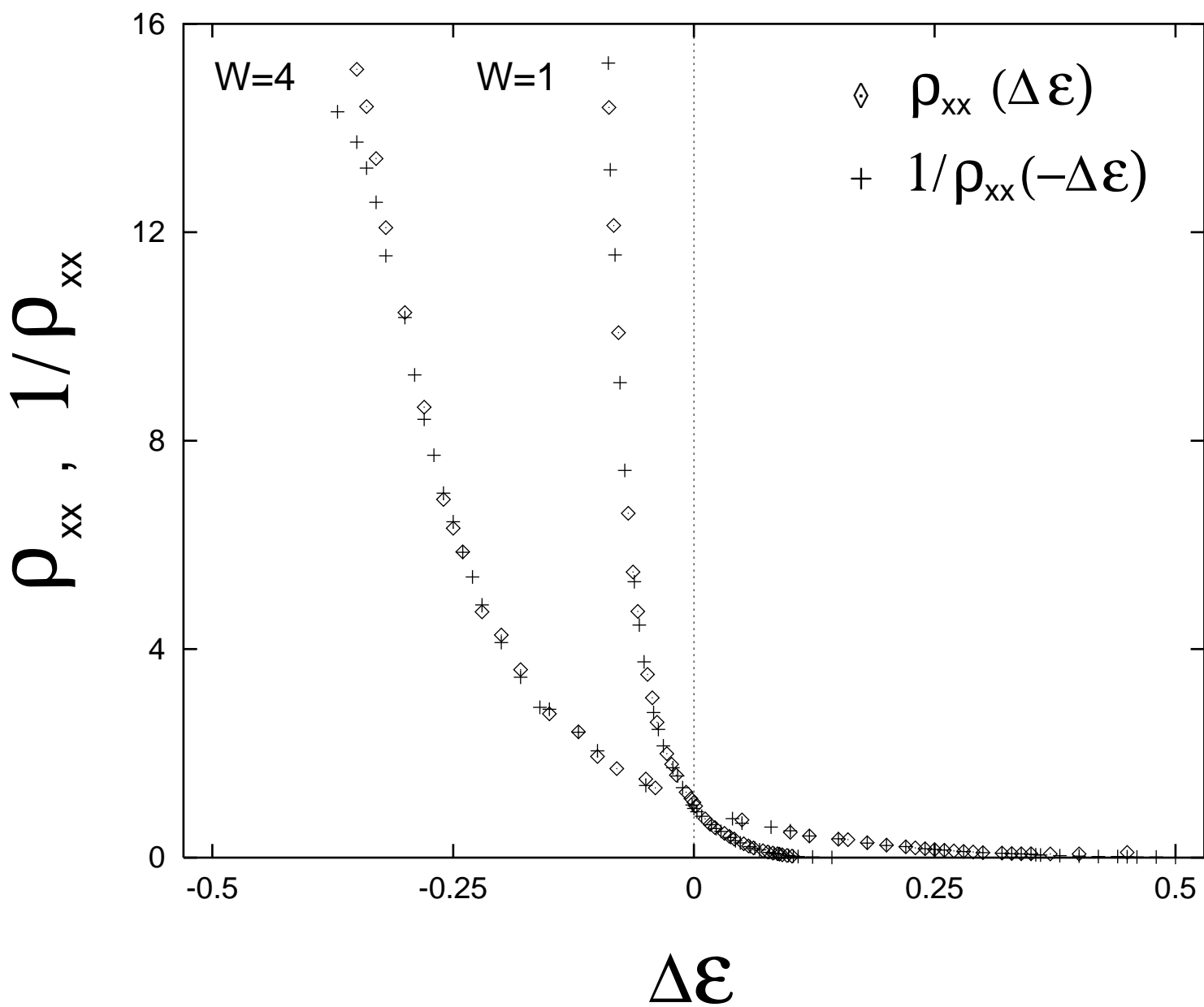


Fig. 2

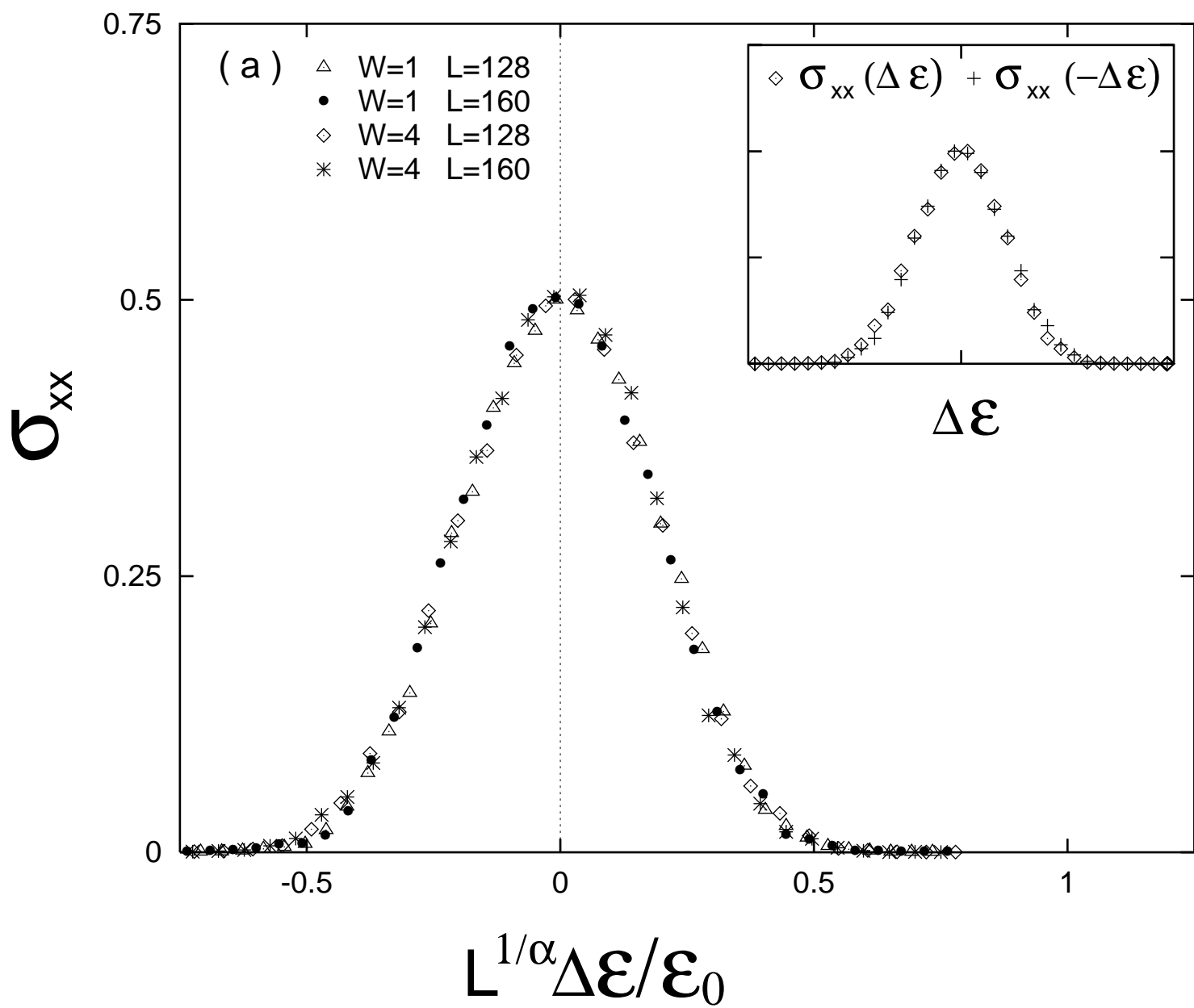


Fig. 3a

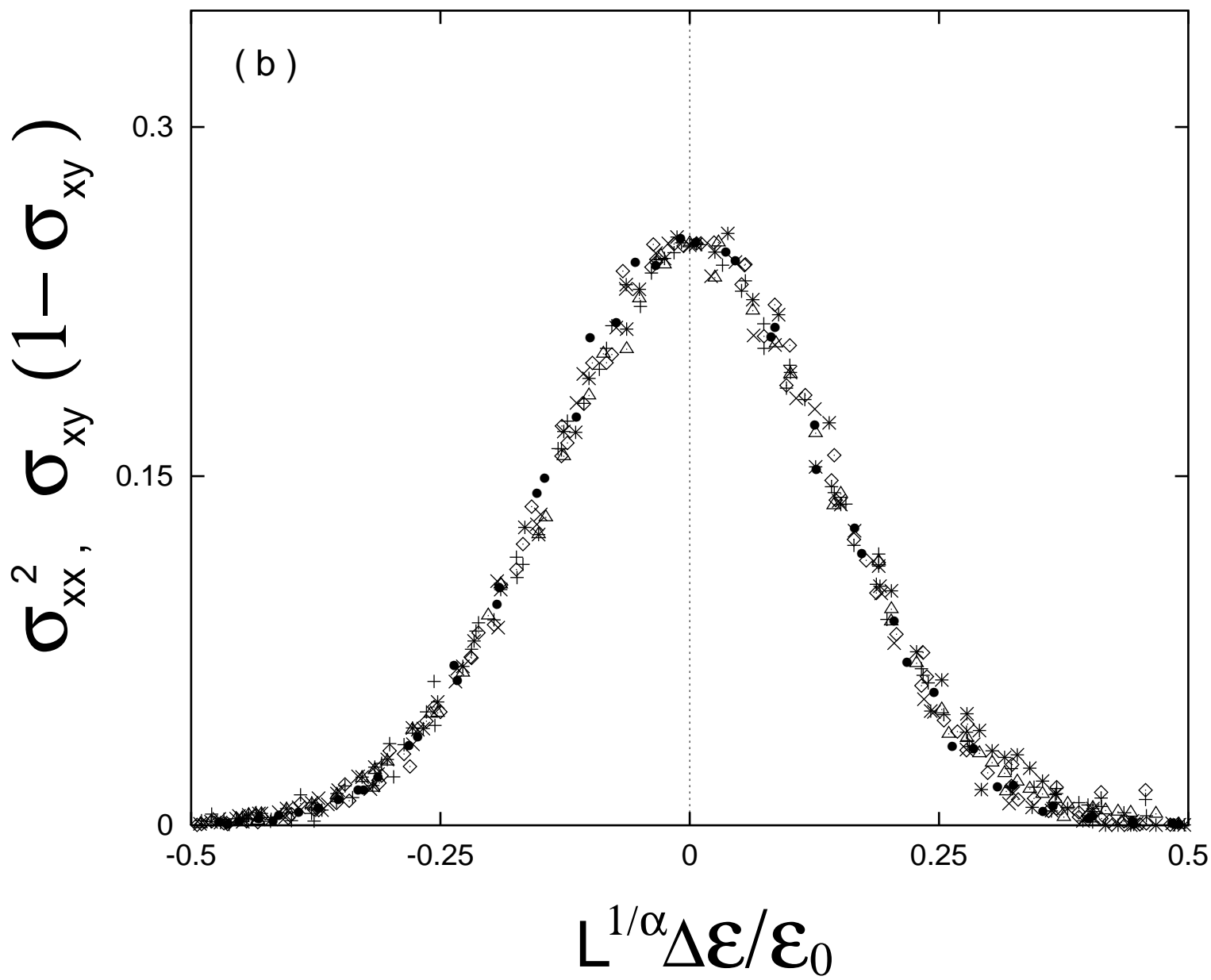


Fig. 3b

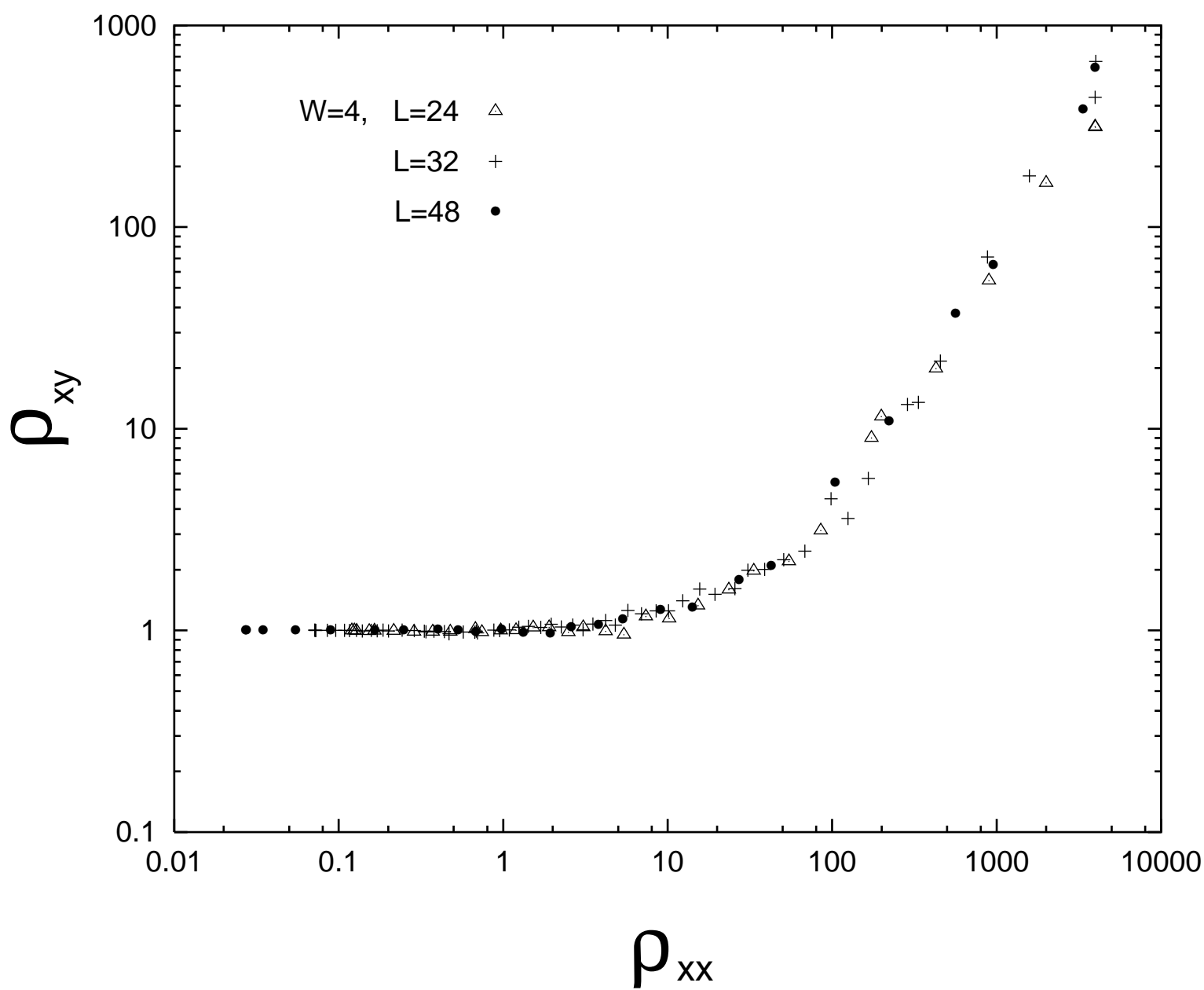


Fig. 4